Cylindrical probe in a flowing plasma

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Letters to the Editor

Cylindrical probe in a flowing plasma

Abstract. The theory developed by Andrews and Swift-Hook for a spherical probe in a flowing plasma can be applied to a cylindrical probe. As with a sphere, a stagnation point forms downstream and there is a small change in floating potential and in ion saturation current. These changes are smaller than for a sphere and opposite in sign; it might just be possible to obtain some indication of the flow velocity from them. Apart from this, existing low pressure, subsonic probe theories are substantially correct numerically.

The theory developed for a spherical probe in a flowing plasma (Andrews and Swift-Hook 1971) can readily be applied to a cylindrical probe. As with a sphere the ion flow can be described (subject to the same assumptions) by the general ion flow equation

$$c^2\nabla \cdot \boldsymbol{v}_\mathrm{i} = \boldsymbol{v}_\mathrm{i} \cdot \nabla(\tfrac{1}{2}\boldsymbol{v}_\mathrm{i}^2).$$

c is the ion speed of sound $(kT_\mathrm{e}/m_\mathrm{i})^{1/2}$, with respect to which all velocities can conviently be normalized. As with the sphere, we divide space into a free stream region (with the flow becoming uniform at infinity, velocity U) and a pre-sheath around the probe (with more or less radial flow); we join the solutions at some convenient radius.

The method of solution in cylindrical polars is very similar to that in spherical polars; the velocity components in the free stream are

$$v_\mathrm{r} = -(B_0/r) + U\cos\theta(1+B_1/r^2)$$
$$v_\theta = -U\sin\theta(1-B_1/r^2)$$

and in the region around the probe

$$v_\mathrm{r} = v_0(r) - U\cos\theta\, C_1 R_1'(r)$$
$$v_\theta = U\sin\theta\, C_1 R_1(r)/r.$$

$v_0(r)$ is the radially symmetric ion flow velocity (normalized with respect to c) which satisfies the zero-order equation

$$(1-v_0^2)\frac{v_0'}{v_0} = -\frac{1}{r}.$$

Thus, $r = -v_0^{-1}\exp\{-\frac{1}{2}(1-v_0^2)\}$ and this is plotted in figure 1. $R_1(r)$ is the solution of the first-order perturbation equation

$$r^2(1-v_0^2)R_1''(r) + r(1-v_0^2)^{-1}R_1' - R_1 = 0$$

with $R_1(1) = 1$ and $R_1'(1) = 0$; it has been found numerically and is also plotted, along with its derivative, in figure 1. The constants B_0, B_1 and C_1 are determined by joining the solutions at some convenient radius s; the results are not very sensitive to

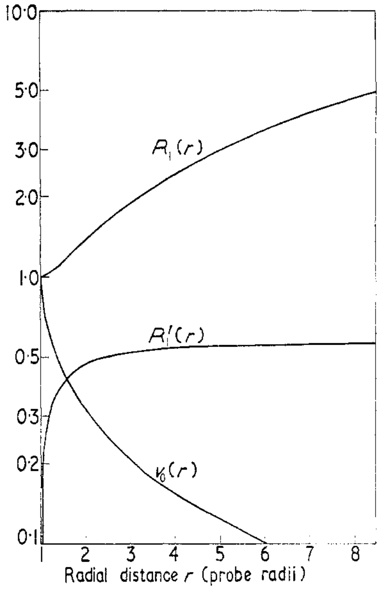

Figure 1. Radial variation of ion flow speed $v_0(r)$ for the static, symmetric case, and velocity perturbation functions $R_1(r)$ and $R_1'(r)$.

the choice of s.

$$B_0 = -sv_0(s)$$

$$B_1 = \frac{-s^2\{R_1(s)-sR_1'(s)\}}{R_1(s)+sR'(s)}$$

$$C_1 = \frac{-2s}{R_1(s)+sR_1'(s)}.$$

The normalized ion flow velocity v, the electrostatic potential V and the plasma density n are then simply given as

$$v = (v_r^2+v_\theta^2)^{1/2}$$

$$eV/kT_e = -\tfrac{1}{2}(v^2-U^2)$$

$$n/n_0 = \exp(eV/kT_e).$$

The ion velocity flow pattern calculated for a free-stream velocity one tenth of the ion speed of sound is shown in figure 2; figure 3 shows the variation of v, V and n on the axis of symmetry for three tenths the ion speed of sound.

These results look very similar to the corresponding ones for a sphere given by Andrews and Swift-Hook (1971). There is a stagnation point behind the probe where the inwards flow to the probe just cancels the free-stream flow. The slower the free-stream flow the further out the stagnation point; it is found to be about $e^{-1/2}/U$ probe radii behind the centre of the probe. This is further away than the $(e^{-1/2}/U)^{1/2}$ which is found to be the approximate distance for the sphere.

The ion current density is nev and the total ion current to the probe per unit length is obtained simply by integrating around the probe. The fractional increase

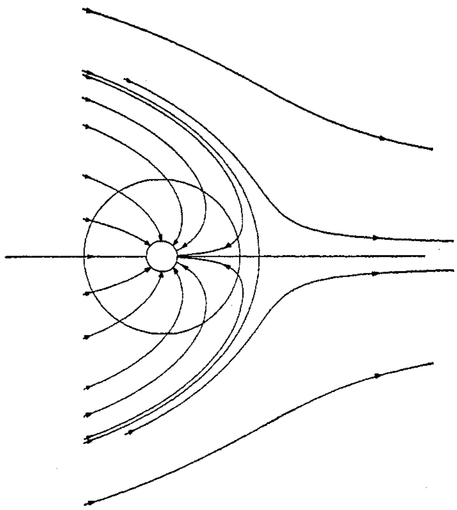

Figure 2. Ion flow around a cylindrical probe, $U = 0{\cdot}1$.

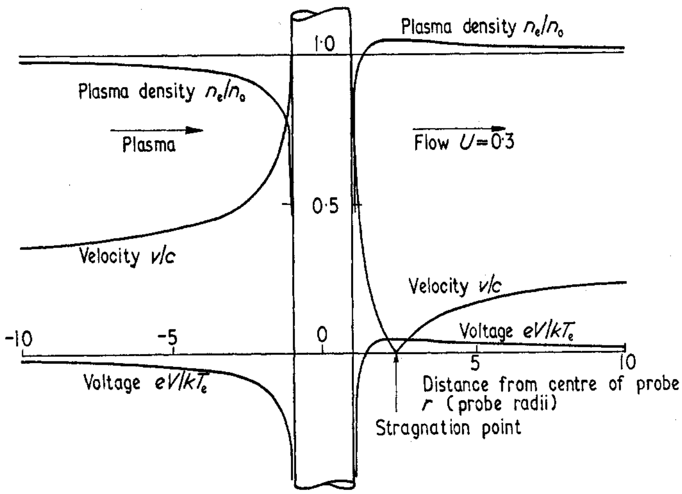

Figure 3. Normalized velocity, voltage and density profiles on axis of symmetry. $U = 0{\cdot}3$.

in ion current due to the flow velocity U (compared with the static case) is found to be

$$\frac{\Delta I_i}{I_i} = F(U) \equiv \left[I_0(X)\exp\left\{\tfrac{1}{2}U^2\left(1-\frac{X}{U^2}\right)\right\}\right]-1$$

where $X = U^2C_1{}^2/4$ and I_0 is the modified Bessel function. $F(U)$ is plotted in figure 4; it is comparable to the corresponding function for a sphere but opposite in sign. For small U, $F(U) \simeq 0{\cdot}115\ U^2$ compared with the corresponding change $-0{\cdot}25\ U^2$ for a

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sphere. The fractional change in the floating potential is then found to be

$$\frac{\Delta V_f}{V_f} = \frac{-\ln\{1+F(U)\}}{\frac{1}{2}\{1+\ln(m_i/2\pi m_e)\}}$$
$$\simeq \frac{0\cdot 30\,F(U)}{1+0\cdot 15\ln W}$$

where W is the atomic weight of the ions' species. As with the sphere, both these changes are small. They are given in figure 4.

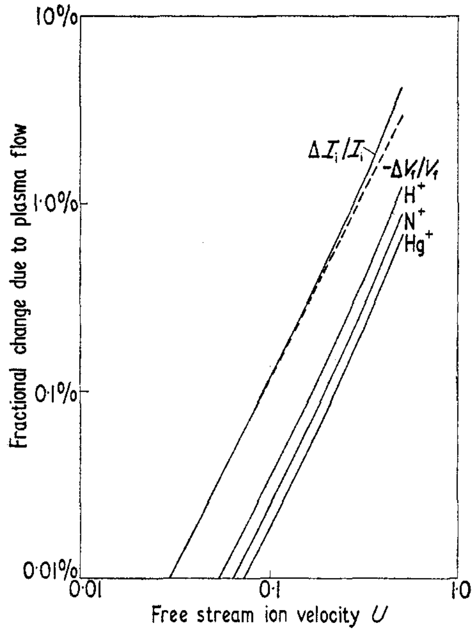

Figure 4. Fractional change in saturation ion current and in floating potential with plasma flow velocity.

It therefore appears that existing low pressure cylindrical probe theories are substantially correct numerically when there is plasma flow. The main differences are that a stagnation point forms downstream and there is a change in floating potential (and in saturated ion current). These changes are small; they are similar to those for a sphere but are opposite in sign.

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